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Uta Lindgren

Introduction: The Situation in 1450

In the period around 1450 there were neither models nor methods for the complete cartographic depiction of the landscape. Scholars who lived in southern German monasteries and at the University of Vienna during the first half of the fifteenth century were involved in intensive work on Ptolemy’s Geography, especially the calculating and collecting of coordinates.1 The scholars’ diverse activities, methods, and instruments were highlighted in two astonishing Munich codices by Fridericus.2 But one cannot produce a large-scale map using coordinates alone. Even though the maps of the Geography came to fascinate the humanists, they were disappointed when they could find neither their place of birth nor their surrounding areas on the maps of ancient Germany. The pleas of Albert Magnus and Roger Bacon in the thirteenth century for high-quality maps could not be fulfilled simply by calculating coordinates.3 The areas between towns with more or less known correct coordinates also had to be filled in on maps (for a reference map, see fig. 19.1).

In order to understand the significance of this absence, we must begin with the larger social and scientific context. Since the eleventh century, mathematical, astronomical, and even geographical learning had been spreading through Europe with the creation of universities. The speed increased considerably after 1400: twenty-eight universities existed in 1400, and eleven more opened over the next hundred years. Popes and sovereigns granted privileges, although they did not participate directly in scientific life. The sons of citizens and the lower nobility supported university life. These were the social groups that essentially profited from the spreading of education, especially by obtaining higher social ranks. The basic university courses, which every scholar was obliged to take, included the artes liberales, particularly mathematics and astronomy. We can trace one aspect of the enormous increase of scientific education in the corresponding increase of scientific manuscripts from the late medieval period that are conserved in libraries.

Another aspect of the period that deserves study was the popularity of anonymous practical manuals with scientific and mathematical content that were first written for the use of merchants (arithmetic), for architects and the building industry (geometry), and for navigators (astronomy). The demand for these practical manuals came from various sources: rich citizens and rich churchmen in growing towns such as Florence, Cologne, London, Paris, and Brugge; merchants; and also sovereigns, who hoped to use them to secure seafaring. These groups of people wanted to embellish their towns with great church buildings and teach their sons the known calculation methods useful for the exchange of merchandise and the art of navigation.

Another impetus for the increase of such knowledge was strong in all social ranks: the desire to know the future by means of astrology. Horoscopes required a good knowledge of the star constellations and the exact time and geographical coordinates of the interested individual’s conception. The latter could not be taken from a map, but had to be determined spontaneously from the actual place. Later a collection of geographical coordinates could be gathered from horoscopes, as Peter Apian...
did early in the sixteenth century. The practical manuals of the later Middle Ages did not equal the significance of the works of Euclid, Boethius, or Ptolemy, but were taught both at the universities and in the town schools. Part of their content became very useful for cartography as the desire for better maps intensified over the course of the fifteenth century. Therefore, we can state that the scientific ground for the cartographic depiction of the landscape was well prepared by scholars. The sovereigns’ interest arose only later in the sixteenth century.4

In 1550, Sebastian Münster wrote: “Everything you measure must be measured by triangles.” Although this sounds like a student’s mnemonic, the question remains: when and where was this basic rule formulated?5 No sources are known. We might consider Vienna, where in 1462–64 Johannes Regiomontanus began a purely mathematical treatise on trigonometric functions as used in astronomy from late Graeco-Roman times.6 Trigonometric functions are, however, only one basis of triangulation. Evidence shows that others, such as Euclid’s Elements, had been available to the Latin-speaking West since about 1120.7 On the other hand, the methods of the agrimensores (Roman land surveyors) would not have been the origin of Münster’s dictum. The original function of the agrimensores was to define the layout and the limits of newly founded towns and military camps, as well as to distribute land to campaign veterans, without seeking to reduce the earth’s surface, especially in mountainous areas, to the geometrical surface of the globe, which was the basis of cartography.

When Johannes Stöffler, from whom we have the first instructions concerning practical geometry for surveyors, began his studies in Ingolstadt in 1472 when the university was newly founded, many masters from Vienna had come to lecture there. Other students who subsequently became cartographers made their way to Vienna, even though Vienna’s initial heyday of studies in mathematics and astronomy was long past and the second had not yet begun.8 Perhaps the University of Vienna ought to enjoy the reputation as the birthplace of practical geometry with its trigonometrical component.

In his “Ludi rerum mathematicarum” (ca. 1445), Leon Battista Alberti describes several procedures of land surveying in much more detail than in his “Descriptio urbis Romae.” After explaining various procedures of practical geometry, such as calculating the height of a tower or the width of a river, Alberti instructs the reader to make a circular instrument at least a braccia wide (60–70 cm), then to divide the edge of the circle into twelve equal parts and each of the twelve parts into four parts, yielding a total of forty-eight parts (called degrees) for the circle, and then to divide each degree into four minutes. Alberti suggests that the instrument be used as follows. The observer selects a flat, high place from which he can see many landmarks, such as campaniles and towers, and lays the instrument flat on the ground. He then measures the

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4. See chapter 26 in this volume.
5. Sebastian Münster, Cosmographiae; oder, Beschreibung aller Länder (Basel: Apud Henrichum Petri, 1550; reprinted [Munich: Kolb], 1992), XXVIII.
Land Surveys, Instruments, and Practitioners in the Renaissance

**Land Surveys**

*Astronomical Methods*

The summary of the various basic cartographic principles explained in the first book of Ptolemy’s *Geography* had fundamental significance until the beginning of modern times. Ptolemy had described the right astronomical methods, but did not use them to determine geographical longitudes. Instead he muddled through with the help of the route distances measured under the emperor Augustus. Although Muslim scholars did not create elaborate maps in the Ptolemaic tradition and using Ptolemaic definitions, their standard of exactness in astronomical observations and the means—that is to say, the instruments—they used first became known to European scholars in the tenth century and thus became models for Europe.

However, methods alone did not create reliable maps. European scholars faced a task that demanded much work, because Europe was densely populated and the landscape structures were of great variety. In addition to the objective difficulties of making maps on the basis of Ptolemy’s methods, efforts were hampered by the fact that interest varied widely during the last four centuries of the Middle Ages and also from one country to another. The first effort to make measured maps—and we know little about how this was done—resulted in the thirteenth century in the portolan charts that were initially limited to the Mediterranean and Black Sea coasts. As a result of Ptolemy’s *Geography*, astronomical observations took absolute priority over geographic observations. The geographic latitude of a location was calculated according to the height of the astronomical north pole. To calculate geographical longitude, it was necessary to carry out several observations of lunar eclipses at different places simultaneously, and all further geometrical observations were adapted to the fixed points so obtained. These geographical coordinates were entered into tables and added to globes and maps. All other observations were not of a geometrical nature.

During the fifteenth century, when the *Geography* was widely distributed, its explanation of methods gained importance, even though that information was somewhat

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sparse. In countries where scholars were active in the field of mathematics, such as Germany, Italy, France, England, Spain, and Portugal, tables of geographic coordinates were compiled and improved. The calculation of latitude through the height of the pole is mathematically sound. Peter Apian explained this method in *Cosmographicus liber*, which became extremely influential through its numerous reprints.\(^\text{11}\) Apian explained the adjustments needed to obtain latitude by observing the height of the midday sun,\(^\text{12}\) a method that Ptolemy had acknowledged needed refinement.\(^\text{13}\) Although Apian’s contemporary, Oronce Fine, proposed a further method involving the introduction of the rising and setting of certain fixed stars, the sun-based methods remained extremely popular.\(^\text{14}\) Sebastian Münster discussed both types of methods, but was not successful in hindering the continued use of the uncorrected sun height method. In succeeding centuries, the value of this method for measuring geographic latitude was often considerably less accurate than for longitude.\(^\text{15}\)

Calculations of longitude based on lunar eclipses were very inexact due to the length of time of an eclipse and the impossibility of calculating exactly when it began and ended. Although some twelfth-century scholars in Latin-speaking countries knew the Islamic astronomical method of calculating the longitudinal difference of two locations by the simultaneous observation of the position of the moon in relation to that of a neighboring fixed star (the lunar distance method), one finds no mention of it in early writings on cosmology during the Renaissance.\(^\text{16}\)

In the earliest editions of the *Cosmographicus liber* of 1524, Apian recommended only the lunar eclipse method for determining longitude,\(^\text{17}\) but he introduced the lunar distance method in editions after 1540. Two observers must determine the difference in their local times before beginning the observations. The second observer can, however, be replaced by a lunar table (ephemerides), as explained in Apian’s text. Oronce Fine covered only the lunar eclipse method in his *De cosmographia* of 1530,\(^\text{18}\) but in the later, separate publication of *De mundi sphaera* he explained the method of comparing the meridional motion of the moon (when the moon passes the meridian of the observer) with the figure in the ephemerides for a central location.\(^\text{19}\) On the other hand, Sebastian Münster described only the lunar eclipse method in his *Cosmography* of 1550, with the interesting variation that the observers should use clocks set to local time on the same evening.\(^\text{20}\)

About the same time (1547), Reiner Gemma (Edelsteine), known generally as Reiner Gemma Frisius (i.e., of Friesland), suggested a new method for calculating longitude when on journeys, namely that of using a portable clock set to the local time of one’s point of departure, which one would compare with the local time at one’s destination.\(^\text{21}\) He pointed out the limited value of this technique, because the mechanical clocks available then were so imprecise that they had to be corrected by comparing them to large water or sand clocks, which were able to run accurately for only a day. Since the eleventh century, local time had been using the astral clock or nocturnal.\(^\text{22}\) Galileo Galilei’s suggested method for longitude determination, using the changing eclipses of Jupiter’s moons, had not been successful in practice.\(^\text{23}\) It was also

\(^{11}\) Peter Apian, *Cosmographicus liber* (Landshtut, 1524); citations here are from the 1540 Antwerp edition, *Petri Apiani Cosmographia*, chap. VII, fol. X.

\(^{12}\) Apian, *Cosmographia*, chap. IX, fol. XI.

\(^{13}\) Ptolemy, *Almagest*, 3.4–9.


\(^{17}\) Apian, *Cosmographia*, fol. XVI.

\(^{18}\) Fine, *De cosmographia*, fol. 145v.


\(^{20}\) Münster, *Cosmographae*, XXXIII.

\(^{21}\) Reiner Gemma Frisius, *De principiis astronomiae et cosmographiae, de qua|e| vsu globi ab eodem editi: Item de orbis divisione, & insula, rebusq[ue] super inuentis* (1530; Paris, 1547), citations from the Antwerp edition (1584), 239. There is disagreement on the proper form of the author’s name; I use Reiner Gemma Frisius.

\(^{22}\) Vernet Ginés, “El nocturlabio,” and Zinner, *Deutsche und niederländische astronomische Instrumente*, 164.

TABLE 19.1 Differences between Longitude and Latitude Values from Four Coordinate Tables and Modern Values

<table>
<thead>
<tr>
<th>Places of the Same Longitude</th>
<th>Modern Values</th>
<th>Oronce Fine, 1541</th>
<th>Johannes Stöffler, 1518</th>
<th>Peter Apian, 1524/1540</th>
<th>Ptolemy’s Geography (Ulm, 1482)</th>
</tr>
</thead>
<tbody>
<tr>
<td>According to Münster, 1550</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basel</td>
<td>7°36’</td>
<td>47°33’</td>
<td>29°45’</td>
<td>47°45’</td>
<td>24°22’</td>
</tr>
<tr>
<td>Strasbourg</td>
<td>7°35’</td>
<td>48°35’</td>
<td>30°15’</td>
<td>48°45’</td>
<td>24°44’</td>
</tr>
<tr>
<td>Kaiserslautern</td>
<td>7°47’</td>
<td>49°27’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Koblenz</td>
<td>7°36’</td>
<td>50°21’</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Münster/Westfalen</td>
<td>7°37’</td>
<td>51°58’</td>
<td>32°00’</td>
<td>52°05’</td>
<td></td>
</tr>
<tr>
<td>Groningen</td>
<td>6°35’</td>
<td>53°13’</td>
<td>29°50’</td>
<td>53°15’</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Münster, 1550</th>
<th>Münster/Westfalen</th>
<th>Groningen</th>
</tr>
</thead>
<tbody>
<tr>
<td>0h 8m 48°</td>
<td>0h 6m 51’</td>
<td>0h 10m 53°</td>
</tr>
<tr>
<td>24°22’</td>
<td>24°44’</td>
<td>22°54’</td>
</tr>
<tr>
<td>47°45’</td>
<td>49°22’</td>
<td>53°16’</td>
</tr>
</tbody>
</table>

an illusion that the Earth’s magnetism could be used to measure geographical longitude. Nevertheless, as more research in the field of geomagnetism was carried out during the second half of the sixteenth century, the hope that it could provide a satisfactory method intensified.²⁴ This idea was still being preached by Athanasius Kircher in the middle of the seventeenth century.²⁵

Peter Apian listed more than fifty towns in Bavaria with their longitude and latitude in an expanded coordinate table.²⁶ Comparison with other tables, in which individual regions were not so fully documented, shows how widely different the measured results could be (table 19.1). Astronomically measured fixed points were the basis for drawing modern maps. The geographic features lying between these points were cartographically fixed using a variety of different methods. These methods drew more and more from geometrical principles.

Terrestrial Methods: Land Surveyors, Geometers, Cartographers

Theoretical Works

The Hellenistic-astronomical method of mapmaking had, at roughly the same time, a terrestrial counterpart among the Romans in the techniques of the agrimensores (field surveyors), although the two methods were never combined before the end of the fifteenth century. The special feature of the techniques of the agrimensores was that they could be used to calculate areas. Geometrically speaking, this method relied on visualizing all areas as combinations of easily constructed squares and rectangles. Triangular areas could not be calculated. The written works of these agrimensores, which also discussed other topics, were known during the Middle Ages and were copied and distributed. In the fourteenth century, the lawyer Bartolo da Sassoferrato used the knowledge of the agrimensores after a disastrous flooding of the Tiber in order to regulate the rights of land possession in the newly formed river valley.²⁷ The most important work on land surveying, “De limitibus constituendis” by Caius Julius Hyginus (ca. A.D. 100), was copied eleven times during the sixteenth century.²⁸ The Maner of Measurynge


²⁶. Apian, Cosmographia, fols. XXXIII r/v.


²⁸. Menso Folkerts and Hubert Busard, Repertorium der mathematische Handschriften (forthcoming).
All Maner of Land, by Richard Benese, appeared in London in 1537, and this again discussed the methods of the agrimensores.29

None of these methods, however, found its way into mapmaking. Indeed, surveyors were content to rely on practical geometry, based on Euclid’s Elements, and to use the principles of triangles therein for triangulation purposes. Euclid was not mentioned in the context of land measurement before the end of the fifteenth century, and it is still not known who was responsible for this shift. Sebastian Münster’s instruction to measure using triangles is found in the middle of this process. Münster, a professor of Hebrew studies, cannot be considered the originator, but he was an effective propagator.

Nobody expressed the idea that triangulation was the decisive method of choice as clearly as Münster had. He could well have learned this from his tutor in Tübingen, Johannes Stöffler, although not all the techniques that he described can be found in Stöffler’s treatise on practical geometry, De geometricis mensurationibus rerum, first printed in Oppenheim in 1513 by Jakob Köbel.30 The individual examples used in the following period varied enormously in these tracts, but this says little about the principles employed. Perhaps the wide range of examples had more to do with the commercial success sought for the booklets. In his work Stöffler explained, using a number of examples, how inaccessible distances could be calculated. One side of a triangle must be measured using a measuring stick (pertica), and angles must be observed. Most of the examples given were based on the similarity of triangles, on proportions or relations, and on the use of the rule of three (Regeldetri). Stöffler also explained a number of examples using a variation of Jacob’s staff where the longstaff and crosspiece are roughly divided, and he said that the position of the surveyor must be carefully chosen to fit into the calculations. The last examples were based on the use of shadow squares with “umbra versa” (vertical shadow) and “umbra recta” (horizontal shadow). These examples assumed knowledge of the cotangent function, even if they did not use the angle values. This had the advantage of avoiding a possible source of error that arose when people measured angles with the simple devices of the time.

In 1522 Jakob Köbel published a German version of Stöffler’s geometry.31 The examples using the shadow square method were not in the first edition; in the second edition, which appeared posthumously in 1536, they were included.32 Köbel’s work popularized the adapted Jacob’s staff and explained that the mirror functioned as a sort of bearing device.

This early modern triangulation got its name from the use of triangles in the surveying of land. It is comprised of a combination of various geometrical components: (1) teachings based on the triangle methods found in Euclid’s Elements; (2) the use of trigonometric functions, with which the sides of a right-angled triangle can be ascertained when one side and one angle are known; (3) practical rules deduced from one or both sources.

Peter Apian employed triangles covering surprisingly large areas.33 He compiled a table from which it was possible to extract the length of a degree of latitude as one traveled away from the equator.34 In his first example of how to calculate the distance between Erfurt in Thüringia and Santiago de Compostela in Galicia, he recommended the use of a globe. From this he was able to read off the coordinates. Apian’s other examples worked along the same lines: the distance between two locations was computed using the known coordinates. However, Apian used plane trigonometry and left the spherical shape out of his equation, calculating the distance between Jerusalem and Nuremberg using a sine table. In each case he dealt with a systematically calculated example, not with an explanation of the method.

Oronce Fine was already aware of Stöffler’s methods of measuring the location of inaccessible points explained in Stöffler’s De geometricis mensurationibus rerum (1513).35 In Fine’s De geometria of 1530, he explained a series of examples using smaller distances in which one side of a triangle could not be measured directly and had to be calculated, for example, the height of a tower visible on the other side of a stretch of water or the depth of a well.36 Geometrically, he used proportions of suitably chosen triangles. In many cases he used the methods of Euclid and to some extent trigonometric functions. At least one distance had to be measured in each case, sometimes up to three distances.37 In some examples, he con-
structured a large geometrical quadrat with sides about one
meter in length as a reference length. Elsewhere he used
his own eye level as a reference. The values of the angles
did not play a role in these examples. In addition to the
geometrical square, Fine employed a geometrical quad-
rant, i.e., a quadrant with an inscribed square and plumb
bob, a Jacob’s staff based on Stöffler’s baculus geometri-
cus, and a suitably positioned mirror.³⁸ Fine left open the
combination in which these measuring and calculating
procedures should be carried out. Later, in his De mundi
sphaera, he devoted a chapter entirely to the construction
of maps, with the example of a map of the French coast
of the Mediterranean Sea measuring about ten by ten
centimeters.³⁹

Gemma published a work in 1533 on land surveying
methods that after 1540 was printed with subsequent edi-
tions of Apian’s Cosmographicus liber.⁴⁰ The aim of the
work was to explain how to construct a map of a partic-
ular area with the aid of land measurements. It contained
information on astronomical as well as terrestrial prin-
ciples and the necessary instruments.⁴¹ Many editions of
the Cosmographicus liber were published, with transla-
tions in Spanish, French, and Flemish. There were also
anonymous editions as well as editions that other authors
had attributed to themselves. The work became by far the
most widespread manual for mapmakers and instru-
mentmakers in the sixteenth and seventeenth centuries.

Gemma’s most important example relates to the deter-
mination of the positions of the towns near Brussels and
Antwerp through the use of angle measurements made
from particular viewpoints (fig. 19.3).⁴² To measure the
angles, Gemma used a compass, a circle divided into
quarters (each further divided into ninety degrees), an al-
dade, and a circular sheet of paper on which to record
the observations for each city.⁴³ On top of each observa-
tion tower, he first used the compass to define the merid-
ian and properly orient the circle; he then used the circle
and the alidade to sight to each distant town and then
drew each bearing onto one sheet of paper whose center
represented the tower. At home, he placed the circular
sheets onto a larger piece of paper, oriented them prop-
erly, and extended the lines of bearings until they inter-
sected, thereby defining the location of each town. He
would have developed if he had used an instrument and
(replaceable) sheets together.

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³⁸. Fine, De geometria, fol. 72.
³⁹. Fine, De mundi sphaera, bk. 5, chap. 6, fol. 53v–54v: “De con-
structione chartarum chorographicarum.” In De Cosmographia (1530),
mapmaking had been treated even more extensively and illustrated by a
rough sketch of the border of France (bk. 5, chap. 7, fols. 154–55).
⁴⁰. Reiner Gemma Frisius, Libellvs de locorum . . . (Paris, 1553), 60v. Photograph
courtesy of the Universiteitsbibliotheek Leiden (20077, A16).
⁴¹. Uta Lindgren, “Johannes de Sacrobosco: Sphera volgare nova-
mente tradatto,” in Copernicus, 221–22.
⁴². Gemma Frisius, Libellvs de locorum, fol. XLVIIIv. For a fuller dis-
cussion, see pp. 1297–98 in this volume.
⁴³. Gemma Frisius, Libellvs de locorum, fol. XLVIIv: “Index cum
perspicillis aut pinnulis.”
which unknown lengths of the sides of a triangle could be ascertained. These can be found in his explanation of the basic terminology of cartography, based on the work of Ptolemy. In the first two examples, the two angles and their connecting base line are measured. These values—at reduced scale—are transferred to paper. The third corner of the triangle is formed by the intersection of the sides. The required distance between this corner point and the base point of the observer can now be taken from the drawing. This graphic method of problem solving was not unusual in other areas during the Middle Ages, but it was rare in land measurement. Although in principle correct, it suffered under Münster from various practical inaccuracies. In the first example, the distances from Offenburg to Basel and Thann were much too great to allow exact bearings to be taken. Münster used an instrument for measuring angles similar to that used by Gemma except that the magnetic compass needle was incorporated. In his second example, the instrument was a *triquetum* (a three-armed instrument also called a *Dreistab*) with two angle measurement devices and a compass. The magnetic compass was used, as by Gemma, to determine the line of meridian, from which the other angles were calculated. This second example worked with proportions.

For his third example, Münster used a geometrical square (the shadow square) that had two sides labeled “umbra recta” and “umbra versa,” just as Stöffler had described in his work, to determine a length common to two triangles without actually measuring any angles (fig. 19.4). In Münster’s example, the unknown length was the width of the river Rhine near Basel; moving along the bank of the river, and taking repeated bearings with the alidade of the quadrat against the umbra recta, he identified those points where the value of the umbra recta was six and

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**FIG. 19.4. GEOMETRICAL QUADRAT AND HEURISTIC MODEL, 1550.** The shadow square (lower left) graphically represents trigonometrical functions; the “umbra recta” stands for the cotangent. To determine the unknown width of the Rhine (AB upper left), Münster determined points C and D along the river bank such that the umbra recta was either six units (half of its full length) or twelve units (equal to its full length), respectively, so that AB was either twice AC or equal to AD.

Size of the original: ca. 31.2 × 19.2 cm. Sebastian Münster, *Cosmographie; oder, Beschreibung aller Länder*. . . (Basel: Apud Henrichum Petri, 1550), 31. Photograph courtesy of the Special Collections Research Center, University of Chicago Library.

Gemma prided himself on the precision of his method, whereby no error could be noticed for distances of up to one hundred German miles (ca. 750 km). When calculating larger distances or greater areas, the problem of establishing the meridian by using a magnetic compass needle could have an effect. The most important factor for accuracy was the measurement of the base line, for which Peter Apian gave instructions.

Sebastian Münster, whose *Cosmography* had been available in German since 1544, explained three ways in which unknown lengths of the sides of a triangle could be ascertained. These can be found in his explanation of the basic terminology of cartography, based on the work of Ptolemy. In the first two examples, the two angles and their connecting base line are measured. These values—at reduced scale—are transferred to paper. The third corner of the triangle is formed by the intersection of the sides. The required distance between this corner point and the base point of the observer can now be taken from the drawing. This graphic method of problem solving was not unusual in other areas during the Middle Ages, but it was rare in land measurement. Although in principle correct, it suffered under Münster from various practical inaccuracies. In the first example, the distances from Offenburg to Basel and Thann were much too great to allow exact bearings to be taken. Münster used an instrument for measuring angles similar to that used by Gemma except that the magnetic compass needle was incorporated. In his second example, the instrument was a *triquetum* (a three-armed instrument also called a *Dreistab*) with two angle measurement devices and a compass. The magnetic compass was used, as by Gemma, to determine the line of meridian, from which the other angles were calculated. This second example worked with proportions.

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twelve units (out of twelve units), such that the distance he had moved along the bank was either half of or equal to the width of the river. Because he did not actually measure any angles or use tables to determine a cotangent for use in a calculation, Münster very cleverly eliminated possible sources of error.

In 1574 Erasmus Reinhold, a doctor and astronomer from Saalfeld and son of the mathematician and astronomer from Wittenberg of the same name, published his Bericht vom Feldmessen und vom Markscheiden.\textsuperscript{48} Besides presenting an introduction to basic calculation with a table of squares for the numbers from one to four thousand for computing their square roots, he also explained commonly recurring examples of land surveying that relied on the similarity of triangles and that could be solved using the rule of three (Regeldetri) and with the calculation of relationships or proportions. This book, like other teaching books, harked back to the field of Euclidian geometry. Reinhold also introduced his readers to the possibilities of using triangulation with the table of sine values supplied by him to solve surveying problems. Here the focus was on the calculation of the sides of a triangle and its area when given measured angles. Reinhold’s required instruments were a measuring stick, a rope for measuring the distances, and a “Compast” angle-measuring instrument—a large circle equipped with a magnetic compass and an alidade. With these, angles could be measured within ten minutes.

All the aforementioned authors incorporated triangulation into the process of map production. Willebrord Snellius had a different objective when he attempted to measure the length of one degree of the great circle along a meridian in his Eratosthenes Batavus.\textsuperscript{49} His base points were Bergen op Zoom and Alkmaar. Somewhere between these, near Leiden, he measured a short base line. Working from this base line, he proceeded to set out a network of triangles on both sides, of which he measured all three sides. The instrument he used was a large quadrant. The method of calculation (using the sine function to determine the lengths of the sides) was not described in his Regeldetri and with the calculation of relationships or proportions. This book, like other teaching books, harked back to the field of Euclidian geometry. Reinhold also introduced his readers to the possibilities of using triangulation with the table of sine values supplied by him to solve surveying problems. Here the focus was on the calculation of the sides of a triangle and its area when given measured angles. Reinhold’s required instruments were a measuring stick, a rope for measuring the distances, and a “Compast” angle-measuring instrument—a large circle equipped with a magnetic compass and an alidade. With these, angles could be measured within ten minutes.

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A few years before his death, Wilhelm Schickard wrote instructions in his Kurtze Anweisung for making maps and explained to simple travelers how they could report information that would keep him and others from taking long and difficult journeys.\textsuperscript{52} The work is a de facto mixture of learned teaching, personal experience, and explanation. As models Schickard mentioned the techniques of Sebastian Münster, Aegidius Tschudi, David Seltzin, Wolfgang Lazius, Georg Sandner, Sebastian von Rotenhau, Johannes Mellinger, and Bartholomäus Scultetus, all of whom had “made maps according to geometrical principles.”\textsuperscript{53} Actually this was true only for Scultetus, and Schickard omitted Philipp Apian.\textsuperscript{54} Schickard incorrectly alleged that Gemma’s method of using a disk divided into 360 degrees was too imprecise.\textsuperscript{55} Instead, he recommended using a simpler device, whereby the circle on a dial would be repeatedly divided until there were ninety-six sections, and then a pointer with an alidade and a magnetic needle compass would be added.\textsuperscript{56} He then applied the method suggested by Gemma (in his Antwerp-Brussels example).

Schickard described a third method based on tables of known distances in and around Tübingen. Using a pair of compasses, he drew circles that intersected at the relevant locations. It was not a precise technique, because the distances in Schickard’s tables were known only in terms of hours. The calculation of coordinates, which Schickard only partially explained, should be used only as a second choice, when the triangulation measuring points are too far apart.\textsuperscript{57}

While Snellius did not consider the construction of maps, and consequently his Eratosthenes Batavus does...
not supply any relevant instructions, Daniel Schwenter explicitly considered surveying in the field, both for civil and military architecture and also for cartography, in his *Ohne einig künstlich geometrisch Instrument*, issued as the third part of Schwenter’s *Geometria practica nova* (1617; 2d ed. 1623), and *Mensula Praetoriana*, issued as the third part of Schwenter’s *Geometria practica nova* (1626). Schwenter’s works contain a greater number of examples of how an inaccessible stretch of land can be calculated using triangulation.

In addition to all the examples mentioned in his *Mensula Praetoriana*, there is also a complete example of measuring the area of a piece of land.58 In order to do this, Schwenter climbed numerous towers, established the meridian using a magnetic compass, and then for every location placed a new sheet of paper on his measuring table and entered the bearing of each important feature in the area. This is the same technique that Gemma taught in his examples using observation points in Brussels and Antwerp, but here it is in expanded form. The joining together of all the sheets was done at home, exactly as Gemma had done it, onto one large sheet, whereby the bearings were extended as far as their points of intersection.

The various works just detailed are characterized by their relative independence from one another and their originality, even if they ultimately employ the same triangulation methods. Many works were based on them.59 Comparing the known maps of that time, we must acknowledge a gap between the theoretical knowledge and education of cartographers and surveyors, on the one hand, and cartographic practice, on the other. This did not mean that there was little demand for maps. On the contrary, Renaissance sovereigns grew more interested in cartographic representations of their realms. For example, the case of Philipp Apian is well documented.60 Duke Christoph of Württemberg, the son of Duke Ulrich, who had called Stöffler to the University of Tübingen, proudly showed a map of his country to his cousin, Duke Albrecht V of Bavaria, when the latter visited him in 1554. Albrecht had been sent to Peter Apian for a scientific education in the company of the latter’s son, Philipp Apian. The experience provoked a great interest in maps in Albrecht, who sent Philipp Apian to Christoph to inspect the map of Württemberg and determine whether he could create a similar map of Bavaria. Deceived by what he had seen, Philipp Apain returned to report that he could easily surpass the Württemberg map—in a “cosmographic manner,” because the Württemberg map was only a painting. As a result, he received the famous commission for his map of Bavaria, which was finished in 1563 with a hand-drawn map measuring 5 × 5 meters and followed in 1568 by a woodcut printing measuring 1.7 × 1.7 meters.

The interest of Renaissance rulers in obtaining good maps was various. The visualization of the sovereign’s heritage often gave him a better knowledge of its range. Knowledge promoted better fiscal control and budget planning. It also served a military purpose, especially allowing realistic planning for specific distances and landscapes. For judicial matters, better knowledge helped clarify possession rights and allowed the sovereign to declare the favored grounds for the most cherished privilege of the sovereign: hunting.

Another factor that contributed to the gap between cartographic theory and execution had to do with the demands of the landscape itself. The outdoor job of a surveyor was hard and dangerous. Philipp Apian was only in his twenties when he performed the measuring work for his map throughout Bavaria, and he chose to do it only in the summer months, while he taught at the University of Ingolstadt in the winter. His brother, Timotheus, who assisted him, died from a riding accident shortly before the end of the work.61 Gerhardus Mercator was already in his fifties when in 1563 he undertook measurements in

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59. Marco Mauro published a work in 1537 that at first glance seems to be an Italian translation of Johannes de Sacrobosco’s “De sphaera” (first half of the thirteenth century) but the enlargement of which reveals that it was heavily influenced by Apian’s *Cosmographicus liber* with appendices by Gemma; see Marco Mauro, *Sphaera volgare novamente tradotto* (Venice: Zanetti, 1537), and Lindgren, “Johannes de Sacrobosco,” 221. Cosimo Bartoli follows Oronce Fine, but also refers to the works of Albrecht Dürer, Gemma Frisius, Philipp Apian, Johannes Stöfler, and Georg von Peuerbach; see Cosimo Bartoli, *Del modo di misurare le distanze, le superficie, i corpi, le piante, le province, le prospettive, & tutte le altre cose terrene, che possono occorrere a gli huomini, secondo le vere regole d’Euclide, & de gli altri piu lodati scrittori* (Venice: Francesco Franceschi Sanese, 1589), with earlier editions published in Venice in 1539 and 1564. See also *La corte il mare i merlanti: La rinascita della Scienza. Editoria e società. Astrologia, magia e alchimia* ([Milan]: Electa Editrice, 1980), and Eberhard Knobloch, “Praktische Geometrie,” in *Maß, Zahl und Gewicht: Mathematik als Schlußglied zu Weltverständnis und Weltbeherrschung*, ed. Meno Folkerts, Eberhard Knobloch, and Karin Reich, exhibition catalog (Weinheim: VCH, Acta Humaniora, 1989), 123–85, esp. 130–31. Giovanni Pomodoro, in *Geometria practica* (Rome: Giovanni Martini, 1603), follows Euclid (Knobloch, “Praktische Geometrie,” 144–45), as do William Bourne (Tyacke and Huddy, *Christopher Saxton, 23, and Taylor, Mathematical Practitioners, 176*) and Leonard Digges (Leonard Digges, *A Geometrical Practise Named Pantometria* [London, 1571]; R. A. Skelton, *Saxton’s Survey of England and Wales: With a Facsimile of Saxton’s Wall-map of 1583* [Amsterdam: Nico Israel, 1974], 24 n. 38; and Taylor, *Mathematical Practitioners, 166–67*). Although from the title it appears that Paul Pfzinzing’s *Methodus Geometrica* (1589) belongs to this group, his method is not mathematical, and the instruments are emphasized in his work. The explanations Pfzinzing gives are remarkably fragmentary.


Lotharingen for his map. After finishing the map, he fell seriously ill, recovering slowly. He never again returned to outdoor jobs, but sent his sons and grandsons instead. Snellius complained bitterly about the hard conditions of outdoor work, and neither Münster nor Schickard ever thought of doing it by themselves. Münster sent out letters to beg for information, and Schickard worked on his booklet as an instruction for others who would do the outdoor job. In the eighteenth century, Peter Anich died from exhaustion after doing such work. The combination of factors—the increasing demand for maps by Renaissance sovereigns and the physical challenges of land measuring work—begin to explain the ways in which cartographic theory was complicated by the practice on the ground.

**Mine Surveying Methods**

Land measurement and staking ownership in the mining industry began above ground in 1300. The oldest mining rights, e.g., those of Kutná Hora (Kuttenberg), clearly laid down the legal significance of the mine surveying process. Map paintings, drawings, or sketches of mines, however, have been preserved only since the sixteenth century. From 1529 there is a panorama-style sketch with mining boundaries from Fichtelberg Mountain in the Erzgebirge (Saxony), and from 1534 a pit ground plan from the vicinity of Kutná Hora (fig. 19.5). All representations from the sixteenth and early seventeenth centuries are exceedingly stylized. Unlike the case of land survey maps, in which distances can be verified on the landscape, modern verification is not possible in the case of mine surveys, because the mines are no longer accessible. The same standards cannot be applied to mine surveying carried out in the early modern era as were applied to land surveying. The methods employed remind one rather of the legal maps that appeared about the same time (discussed later).

In Schwenter’s *Ohne einig künstlich geometrisch Instrument*, geometrical techniques are included for two mine surveying methods, which combine measurements on the slope and underground. Again, Schwenter started with similar triangles and referred only to Euclid. The two examples were designed not for the production of maps, but as aids to making decisions in connection with digging pits or tunnels (fig. 19.6).

Georg Agricola studied theology and later medicine, and in 1527 became the town physician of Jáchymov (St. Joachimsthal), the center of Saxony’s silver mining district. From 1533 Agricola was a four-time mayor of Chemnitz. He published several books on mineralogy and geology. Agricola’s chief work, *Vom Bergwerck*, on Saxony’s mining and geology, has an illustration of an...
iron angle bracket and a quadrant mounted inside a circular frame with a pendulum pointer (fig. 19.7). It is depicted in the landscape in such a way that one surmises it must have been used on the slope. The text makes no mention of this; it mentions only a magnetic compass, string, and writing utensils for use underground. Agricola does not explain any of the methods used.

Erasmus Reinhold’s 1574 *Bericht vom Feldmessen und vom Markscheiden* has several examples using two separate systems: first, using similar triangles according to Euclid, and second, using sine tables or umbra tables. He therefore suggested the same methods used in land surveying. Besides the measuring rods and a water level, he employed instruments such as a quadrant with an integrated compass, a sighting tube (instead of the alidade), and a “pit level,” i.e., a semicircle divided into degrees and fitted with a plumb line. Due to the limited lighting available underground, how far triangulation could be carried out in practice is questionable. This is also the case for the use of the “mathematical measuring box,” a mathematical precision instrument constructed by Tobias Volckmer (1589) of the Dresden Kunstкамmer, for which a set of operating instructions was written in 1591. This included instructions for use in the mine surveying business.

**Town and City Surveys**

Geometrical techniques had been used since the sixteenth century for town and city surveys. Augustin Hirschvogel employed the methods of Gemma for his 1547 plan of Vienna, and it is suspected that Johann van der Corput

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used the same for Duisburg. How exactly Jakob Sandtner created his extremely accurate town models in relief remains unclear, but their quality strongly suggests the use of triangulation. He made five such models of Bavarian towns between 1568 and 1574. Sandtner insisted on being given written authorization to make observations (from high places) in the fortified city of Ingolstadt.

**INSTRUMENTATION EMPLOYED**

The instruments preserved in the Kunstkammern collections in places such as Dresden, Florence, and Cassel astound the modern observer with their versatility and the preciseness of their manufacture. To take these instruments as the starting point from which to reconstruct their function would pose considerable difficulty. The analysis becomes somewhat easier and nearer to the surveyors’ reality if we begin with the tracts on surveying and the instruments illustrated therein. However, in principle, no method of land surveying relied on one particular instrument.

The instruments used in land surveying were employed in carrying out three tasks: measuring time, measuring distances, and measuring angles, including determining the bearings of a position. Other aids were the magnetic compass, the plane table, the ephemerides, and the surveyors’ assistants.

**MEASURING TIME**

In order to determine longitudinal coordinates, it was necessary to establish the local time of the observer. To do this, Islamic scholars developed an instrument that employed the daily orbit of Ursa Minor and Major around the north pole. This astral clock, also known as the nocturnal, had been used since the end of the eleventh century in the Christian West. This is one of the instruments that can be found illustrated on the title page of Peter Apian’s book of instruments (1533). Apian also depicted and described it in his *Cosmographicus liber* (fig. 19.8; and fig. 19.9 for another example).

In 1547, Gemma pointed out the fundamental significance of portable mechanical clocks, i.e., pocket watches, for determining longitude. In order to be of any use, they had to be continually checked due to their inefficiency, so their implementation in practice was more or less illusory. The importance of precision can be easily seen when one considers that the heavens appear to move at a speed of fifteen degrees per hour, i.e., one degree in four minutes, and one minute in four seconds. More reliable, but unfortunately not always portable, were water and sand clocks. In order for the correct time to be read from a sundial, an attempt was made to take into account on the dial itself the seasonal differences in the movement...
of the sun (figs. 19.10 and 19.11). Assuming that the movement of the sun had been sufficiently researched, the dial still presented a difficult task for the mathematician. As a consequence of these problems, precise measuring of time using the sundial seldom occurred in a surveying context.

MEASURING DISTANCES

In the simplest cases, Jakob Köbel and Peter Apian began by explaining how to measure distance by paces, and Sebastian Münster suggested that the time taken to cover his base from Basel to Thann be converted into miles. No particularly exact results could be thus attained for the purposes of land measurement. Ropes, used for measuring distances of several meters, were not a good measuring instrument due to their sag. Chains were not much better, because slack caused by their great weight could not be completely avoided even when several strong assistants were at work. Much more accurate results could be obtained by using measuring poles, although in those days they were made of wood, which changed slightly in length depending on the weather. This was a minor cause of error. For short distances, one used a measuring stick or ruler, sometimes made of metal. Ease of use and relatively exact measurements over a distance of several miles were possible with odometers. These mechanical measuring devices on wagons go back to late antiquity and the engineer and architect Vitruvius Pollio. August I of Saxony received an odometer of fire-gilded brass, a precisely manufactured device mounted on a traveling coach, from Christoph Trechslar the Elder in 1584. In order to measure a useful stretch for triangulation purposes, the carriage could not drive along any street but had to travel a long, straight stretch or across large (presumably previously harvested) fields. The possibilities for such a precise device to serve surveying purposes were therefore seriously limited. It could obviously be put to other uses, however, for example, to measure the distance between two or more towns.


82. Wunderlich, *Kursächsische Feldmesskunst*, 60–63.
FIG. 19.11. SUNDIAL IN THE FORM OF A POPLAR LEAF, 1533. The use of this sundial, illustrated by Apian, required an adjustable, three-part gnomon to take into account latitude and month.

Size of the original: 31 × 24.5 cm. Peter Apian, Instrument Buch (Ingolstadt, 1533). Photograph courtesy of the Beinecke Rare Book and Manuscript Library, Yale University, New Haven (Shelfmark QB83 A63+).
MEASURING ANGLES

Traditional Instruments

Traditionally, angle measurement was significant only in the study of astronomy. This changed during the fifteenth century. Its use in land surveying has been described in teaching manuals since the sixteenth century.

When Peter Apian published his *Instrument Buch* in 1533, the most important instruments, namely the quadrant, Jacob’s staff (or cross staff), and geometrical quadrat, were as useful for astronomy and navigation as for land surveying. That dual use was also practiced by the successful authors and instrumentmakers who resided in Nuremberg, e.g., Georg Hartmann and Johannes Schöner. But early land measurement, like navigation, did not need as exact and complicated instruments as astronomy. Until the middle of the century, the land surveyors, with their own special methods, preferred to make their own instruments. In some cases, weight was taken into consideration. Consequently, surveyors who did not want to tax themselves during their fieldwork preferred light instruments, i.e., instruments with staffs made of wood.

The quadrant is a quarter of a circle fitted with an alidade on one of the arms and with a plumb bob that hangs from the center point of the circle. Even Ptolemy described a quadrant. In the Middle Ages this was preferred to the astrolabe for certain areas of work because its graduated scale was larger in proportion to a whole circle instrument. The very large wall quadrants of Ulugh Beg in Samarkand and Tycho Brahe in Uraniborg are well known. In order to calculate the distance of the moon from one of the fixed stars, one needed only the scale of degrees. In land measurement, the quadrant was especially appropriate for the determination of heights, as the plumb showed the angle, while one could better observe the horizontal angle using the quadrant fitted with a pointer.

The Jacob’s staff was designed to measure angles between the horizon and a star. The idea can probably be traced back to Hipparchus (190–120 B.C.), but it was improved in the Middle Ages by Levi ben Gerson (1288–1344). Very clear instructions for its production and use can be found in Peter Apian’s *Cosmographicus liber*. With these, anyone could make and use his own. However, the manufacture of the finely divided scale required a considerable amount of skill. This instrument had no other function than that of angle measurement. Due to its simple construction and ease of use, the Jacob’s staff was widely used. Even simpler was Fine’s *baculus geometricus*, with its coarse scale.

The geometrical quadrat had been used since the eleventh century in the measurement of angles. One can see it being employed in this function on the title page of Peter Apian’s *Instrument Buch*. In describing it, Apian made special mention of its use in the calculation of distances. As he used it, it was simply a square frame, while other authors had a square wooden board. At least two sides were divided into as many units as possible; Apian divided his into one thousand units. In one corner a movable arm, the *regula*, was fixed. This strip and at least one of the sides was fitted with an alidade. These devices were produced with a side length of up to approximately one meter.

Determining the Bearings of a Position

In addition to calculating angles and measuring distances, this geometrical square was also used in combination with a magnetic compass for taking bearings of observations in land surveying. Using the methods applied by Gemma and the plane table, it was then no longer necessary to calculate any angles. Finally, a special function of the geometrical quadrat, the shadow square, could also be used for measuring methods based on the umbra recta (cotangent) function. This is explained clearly by Sebastian Münster in his *Cosmography* (fig. 19.4). Very often the theoretical approach cannot be easily recognized, because it was usual to calculate the examples without explaining the mathematical principles behind them.

The Brunswick (Braunschweig) instrumentmaker Tobias Volckmer manufactured a *quadratum geometricum*...
Because of its weight, it was equipped with a tripod support. An attempt was made to convert the quadrant to a universal instrument by the addition of other, sometimes removable, instruments.

During the Renaissance the *triquetum* (also known as a *Dreistab*) was widely used in surveying, although it was less appreciated in astronomy and navigation. Its tradition goes back to classical times. In the early modern period, it was regularly modified by adding full circles to the pivot point for the calculation of angles and a magnetic compass on the side for obtaining the north-south bearing. All three arms, which were pivoted at two points,

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97. The Medici Stanza delle Matematiche in Florence obtained the instrument. Miniati, *Museo di storia*, 27 and 29, pl. 27.
98. Schmidt, *Geschichte der geodätischen Instrumente*, 193, 369–81, pl. XXIV, figs. 4 and 7. Kiely devoted some research to this instrument, but supposed a special triangulation function. The study of the treatises on practical geometry shows that none of the instruments was strictly tied to one mathematical method; see Kiely, *Surveying Instruments*, 220–24.
were divided linearly. The device was especially versatile in the Euclidean-based triangulation seen in the work of Sebastian Münster, Daniel Schwenter, and others (figs. 19.13 and 19.14).

The special characteristic of Schickard’s *triquetum* was the joining together of the three arms to form an equilateral triangle. At the ends of the staffs Schickard fixed alidades, and he placed movable alidades along the sides. He wanted to divide the necessary scale with values found in the table of tangents. However, he forgot to mention that he needed to draw the height of a triangle as a guide line, because only from this was he able to determine the tangent value. Then he was able to continue the line to the side of his triangle. Whether this brought distinct advantages in practice, where only the most accurate angle calculation would lead to good cartographic results, has not been proven, because this device did not find wide use. Schickard claimed that his *triquetum* was lighter than an equally large circular disk.

Another tradition, also traced back to classical times, used two pivoting arms and was described by Münster and Leonhard Zubler. A precisely constructed instrument with two staffs (Zweistab) by Lucas Brunn and Christoph Trechsel the Elder from 1609 was owned by the Dresden Kunstkammer. This instrument was equipped with a finely divided scale and a micrometer slide for exact settings.

A further instrument with classical roots, which in modern times went under the name theodolite, had evidently fascinated the instrumentmakers since the fifteenth century without actually being used practically

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This device could be turned in the horizontal plane as well as in the vertical by means of cogged wheels and was described for the first time by Heron of Alexandria (ca. 100 B.C.) under the name dioptra. In the fifteenth century it became known as the torquetum due to its rotating ability, from which it came to be known as the Türkengerät (Turkish device) in colloquial German. Martin Waldseemüller named it the polimetrum because of its versatility. It first became a widely used precision instrument in the eighteenth century, when a telescope was mounted instead of the alidade.

Innovations (Instrumenta Nova)

Although it is difficult to identify a clear point at which there was a turn from traditional to modern instruments, the inventiveness and willingness to develop ideas in the instrument market during the Renaissance were astounding, even though many alterations hailed as innovations changed neither the construction nor the application of traditional instruments. Consider, for example, the increased size of the sector of the circle. Philipp Apian designed an instrument, the Triens, whose scale was larger than that of a quadrant. Thomas Geminus invented a combination quadrant that one could double in size and, by means of a wooden connecting piece, could extend to over 180 degrees. The tendency, however, was to attempt to reduce the whole circle to a useful minimum (the angle and, therefore, the scale): the sextant down to 60 degrees and the octant to 45 degrees. These tools, which from the last quarter of the sixteenth century (an early example comes from Jost Bürgi in Cassel) had been successful in navigation, were also used in land measurement.

The ring instruments represented a fundamental innovation, because their smaller dimensions made them useful as traveling instruments (fig. 19.16). Gemma’s Usus annuli astronomici marks the beginning of this development, which was successful well into the eighteenth century. They served as sun clocks as well as surveying instruments, but they were usually limited in their accuracy.

102. Zinner, Deutsche und niederländische astronomische Instrumente, 191–92. The term “theodolit” probably comes from Leonard Digges, who was the first Englishman to explain an instrument similar to that of Heron, Regiomontanus, and Waldseemüller, with the difference that his instrument lacked gears and could not be moved into all positions, including slanting. See Taylor, Mathematical Practitioners, 167; Kiely, Surveying Instruments, 180–84; and Richeson, English Land Measuring, 61–64.


105. See Zinner, Deutsche und niederländische astronomische Instrumente, 163–64, under the instruments for the measurement of time.

106. Conserved now in the Museo di Storia della Scienza, Florence; see Miniati, Museo di storia, 32 and 33, pl. 55, and Turner, Elizabethan Instrument Makers, 12–23.

107. Zinner, Deutsche und niederländische astronomische Instrumente, 268–76.

108. Miniati, Museo di storia, 14 and 15, pl. 75.

109. Gemma’s Usus annuli astronomici first appeared in Petri Apiani Cosmographia, per Gemmam Phrysium (Antwerp, 1539). The invention was attributed to Johannes Stabius; see Zinner, Deutsche und niederländische astronomische Instrumente, 539. According to Taylor, the ring instrument was also a favorite of some English surveyors and
Innovations can also be seen in the area of the development of sighting tubes to replace the alidade. Considering the wealth of ideas that were applied to instruments, it is surprising that sighting was not recognized as a problem much earlier. Although sighting tubes were used on instruments for mining surveying in the sixteenth century, it was not until 1555 that they were described for use in land surveying. This was Abel Foullon’s holomètre, a tablelike device with a sighting tube (figs. 19.17 and 19.18) that inspired Philippe Danfrie to construct his graphomètre in 1597. Other types of sighting aids were target disks and signal fires, which at the beginning of the seventeenth century were used in Switzerland to make the needed points more easily recognizable in the distance. Surveyors found plenty of targets to aim at in Germany’s landscape, including church steeples and other towers.

In an illustration by Jean de Merliers from 1575 showing the process of measuring with the aid of a chain, one can see in the foreground a sighting instrument that made it easier to take bearings of a distant point (fig. 19.19). A wooden quadrant stands horizontally at the eye level of the surveyor. Four straight grooves, forty-five degrees

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Footnotes:

110. Abel Foullon, Descriptione, e uso dell’holometro (Paris, 1555; Venice, 1564 and 1584), and La corte il mare i mercanti, 146.

111. Philippe Danfrie, Déclaration de l’usage du graphomètre (Paris, 1597); La corte il mare i mercanti, 146; Turner, Early Scientific Instruments, 253; and Koenraad van Cleempoel, A Catalogue Raisonné of Scientific Instruments from the Louvain School, 1530–1600 (Turnhout: Brepols, 2002), 205.


apart, have been cut into the wood. Inscribed inside the quadrant is a circle, at the middle point of which the four grooves meet (why there must be four such grooves instead of only one or two is not clear). This instrument guided the observer’s line of sight in a similar way as did the sighting tube with which Reinhold had equipped his mountain quadrant instead of using the normal “sighting hole” (alidade).114 Such sighting aids are missing from most of the instruments.

The Magnetic Compass

Sebastian Münster used the magnetic compass at the beginning of his surveying discourses due to its circular form as a measuring tool for angles, but later he believed the larger semicircular or circular disks were better suited to this task. Even so, he incorporated a magnetic compass into all of the instruments he used. This served to identify the north-south direction from which all other angles could then be measured. This method can lead to problems due to magnetic declination. For bearings that lie close to each other, this does not play a role, because the angles are always based on the same direction line, even if this is not exactly north-south. But when this method is used, as recommended with Gemma and Schwenter’s plane table, over a greater region without any checking taking place—and we must assume that this happened—the change in declination can have consequences. Nothing about this is mentioned in writings on surveying, even though variations in declination had been known from the fourteenth century.115

Disregarding such variations for a moment, in the sixteenth century there was speculation that one could ultimate...
mately find a connection between magnetic declination and geographical longitude that would make the determination of longitude easier. This speculation can first be read in Giovanni Battista Della Porta’s book *De magnete*, although Della Porta did not claim to be the originator of this idea.116 Della Porta, a humanist scholar, founded an academy in Naples after extended travels. *De magnete* was part of his chief work, *Magia naturalis*, which was first published as a small booklet in 1557, then as a longer version in 1589. Most of *Magia naturalis* has alchemistic and magical content, but the long chapter on magnetism is an interesting statement of contemporary knowledge enriched with some popular jokes. The idea about magnetic declination and longitude was amplified by Athanasius Kircher, who evidently also taught this in Rome.117 Dutch mariners, especially the sailors of North Atlantic waters, had recorded magnetic declinations in tables quite early. For example, one can find these aids printed by Adriaan Metius, a physician and mathematician who studied in Franeker, Leiden, and Denmark with Tycho Brahe and later became a mathematics professor at the University of Franeker in 1598, but also by Kircher.118

Sebastian Cabot wrote a commentary about the magnetic declination in the North Atlantic to accompany a depiction of America.119 On some Portuguese world maps of the sixteenth century, a slanting bar is shown that indicates that the magnetic needle in the North Atlantic no longer points approximately north, but northwest.120

**The Plane Table**

The surveyor’s plane table allowed angles of bearings or directions of bearings to be added directly onto paper. Daniel Schwenter was the first to describe this as a geometrical instrument; he called it the *geometrisches Tischlein* or *mensula Praetoriana*, because he thought it had been invented by his teacher and predecessor at the University of Altdorf, the mathematician Johannes Prätorius, about 1590 (fig. 19.20).121 It was a portable device consisting of a wooden quadrat with a robust frame mounted on a tripod. Set in the quadrat was a magnetic compass, and on one of the frame sides a movable ruler had been mounted on a rail. In addition, a geometrical drawing device was included, but this was of course not fixed in position. The surface area of the quadrat was covered with paper on which the bearings were drawn.

Without naming Prätorius as inventor and without giving the device a name, Paul Pfinzing, a Nuremberg patrician, described the measuring table as early as 1589 and

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praised its application.\textsuperscript{122} Often more informative than Pfinzing’s words, however, are his illustrations (figs. 19.21 and 19.22). One wonders about his verbal inaccuracies, which possibly arose from a lack of rhetorical skills but certainly not from a lack of know-how. The work was reprinted, unaltered, in 1598. There is another description of the plane table from the same time by Cyprian Lucar in \textit{A Treatise Named Lucar Solace} (1590).\textsuperscript{123}

In 1607 the Swiss engineer Leonhard Zubler published a tract on the construction and application of the plane table, which he named \textit{Instrumentum Chorographicum}.\textsuperscript{124} He ascribed the invention of the plane table to his compatriot mathematician Philipp Eberhard.

The beginnings of the plane table seem to go further back, to the middle of the sixteenth century. No one claimed to be the table’s inventor, but it seemed to have had several fathers. Gemma made use of a similar device without noting that this was a new and very practical instrument. Maybe he got the idea from Alberti’s drawing device, which he used in his own ways. Likewise, Leonard Digges used the back of his “topographical instrument” to plot out the observed bearings as if it were a plane


\textsuperscript{123} Richeson, \textit{English Land Measuring}, 77–81; Taylor, \textit{Mathematical Practitioners}, 328 and 330; and Kiely, \textit{Surveying Instruments}, 230–34. Kiely did not know the German tradition, but assumed that the practical use of the plane table was infrequent in England. On the Continent, the instrument was a great success for many centuries.

\textsuperscript{124} Dürst, \textit{Philipp Eberhard}, 22–24.
How Surveyors or Mapmakers Obtained Their Knowledge

This section introduces the scientific backgrounds of some key authors who wrote on practical geometry and surveying, although some of them were also active in theoretical debates. Knowledge about the ability to determine geographic coordinates astronomically remained at the level of late medieval astronomy; it could be gained at most universities where the Quadrivium was taught. The fundamentals of astronomy being taught came out of Johannes de Sacrobosco’s Sphaera mundi, which itself went no further than book two of the Natural History of Pliny the Elder. This explains the fixation, for example, on the antiquated and inappropriate method for determining longitude using the eclipses of the moon. This is curious, because in other fields of study authors usually prided themselves on knowledge of the latest discoveries, which would have been the lunar distance method for this problem.

Johannes Stöffler wrote his short tract on practical geometry more than thirty years after he matriculated at the newly founded University of Ingolstadt in 1472. Stöffler studied for three years, and a short time later took over as parish priest in his hometown, Justingen (near Blaubeuren), where he remained for over thirty years. When Stöffler wrote his tract on geometry in 1511, he had just become—on the urging of Duke Ulrich of Württemberg—professor of mathematics at Tübingen. In Stöffler, Sebastian Münster found at Tübingen a teacher who taught practical geometry and, more particularly, land surveying. In those days, students had to do their bachelor’s degree in the faculty of arts before being allowed to change to another faculty. Arithmetic, geometry, and astronomy (along with music) were part of the basic

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126. Augustin Hirschvogel’s instructions do not have a title; they exist in several manuscripts, for example, Vienna, Österreichische Nationalbibliothek, Cod. 10.690.
127. Fischer, “Hirschvogels Stadtplan von Wien,” 8. Hirschvogel reportedly used six quadrants, and this made a difference for the devices used by Alberti, Gemma, Pfanzing, Schwenter, and those who advocated using the plane table.
128. Pfanzing, Methodus Geometrica, fig. preceding XIX.
curriculum of the Quadrivium. Exactly how thoroughly the mathematical-astronomical studies were covered depended largely on the teacher. But none of those who began teaching geometry at Ingolstadt are known by name.

Stöffler’s *De geometricis mensurationibus rerum* covers only a very small part of the *Elements* of Euclid, but it deals with practical examples that a land surveyor, architect, or engineer might need. Specifically, the introduction of trigonometry had evidently been completed by the time Stöffler studied at Ingolstadt. Later he liked to look back on his period of study at Ingolstadt, but he had not shown any particular interest in mathematics in those days. The tutors for the faculty of arts were masters recruited from Vienna, where in the previous decades a famous mathematical-astronomical school had developed to which Georg von Peuerbach and Johannes Regiomontanus belonged.133

Jakob Köbel was a humanistically educated publisher and politician in Oppenheim on the Rhine.134 He had achieved a bachelor’s degree in law at Heidelberg, and in 1490 studied mathematics and astronomy at the University of Cracow. In 1492 he returned to the Palatinate and printed a number of Stöffler’s works. His practical geometry is not his own work; he merely published Stöffler’s work in German.

Peter Apian, from Leisnig in Saxony, was forty-three years younger than Stöffler.135 He had studied from 1516 to 1519 at the University of Leipzig and the following two years in Vienna, where in 1521 he was awarded his bachelor’s degree. Leipzig was the first choice for students from Saxony (including Regiomontanus), but Vienna was always attractive. Apian’s mentors have not been identified, although his mathematical work is linked to the Viennese School and especially to Regiomontanus’s writings on trigonometry.136 From 1526 until the end of his life, he occupied the chair of mathematics and astronomy at the University of Ingolstadt, where he transferred and continued the printing office that he had started at Landshut.

In 1513 Johann Scheubel made his way from Kirchheim unter Teck to study in Vienna.137 Twenty years later, he enrolled at Leipzig and obtained his bachelor’s degree. In 1535 he matriculated at Tübingen, where in 1540, he was awarded a master’s degree in 1540, proceeded to teach courses on geometry, and was made *Euclidis professor ordinarius* in 1550. A sketch of the boundaries of the city of Esslingen (1556–57) and a map of Württemberg (1558-59) are ascribed to him, neither of which was based on astronomical-geometrical principles.

Oronce Fine was one year older than Apian.138 He learned his first lessons in mathematics from his father, a doctor in Briançon. After his father’s early death, he was accepted at the Collège de Navarre in Paris. Due to his specialized interest in mathematics and astronomy, he soon left and matriculated at the University of Paris, where he obtained a master’s degree in the faculty of arts. From 1518 until 1524, he was in prison, probably for a failed horoscope. From 1525 until his death, he occupied the chair of mathematics in Paris. Although his surveying instructions are similar to Stöffler’s, it is not clear whether Stöffler’s work was part of the curriculum at the Paris University or whether Fine came across Stöffler through his own reading.

Reiner Gemma Frisius went to school in Groningen, studied at the University of Louvain under unknown teachers, and gained his doctorate in medicine.139 He then taught at Louvain as a professor in the medical faculty. He instructed Gerardus Mercator in the construction of astronomical instruments and globes as early as the 1530s140 and taught the English cosmographer John Dee.141

Interest in mathematical-astronomical questions must have been unusually great in the Low Countries toward the beginning of the sixteenth century. Gemma was only sixteen years old when Apian’s *Cosmographicus liber* (1524) first appeared in Landshut. The fact that in the same year it was republished in Antwerp shows how well it was received there. From 1529 Apian’s work appeared with Gemma’s commentaries and supplementary writings. From the

134. Benzing, Jakob Köbel.
second half of the sixteenth century, interest in nautical methods and aids was more widespread in the Low Countries, which in the areas of mathematics and astronomy was often based on the same principles as cartography. Gemma’s interest was not in delving into the mathematical foundations; his contribution inspired methods and instruments for the practice of terrestrial surveying.

Another Dutch university that emphasized scientific research and teaching was that at Leiden. Willembrord Snellius was there as successor to his father, Rudolf, in the chair of mathematics.142 Adriaen Metius studied in Leiden and in Franeker before he, too, became professor of mathematics in Franeker in 1598.143 His manuals, which were used in Altdorf among other places, were still being translated and distributed in the eighteenth century.144 In addition to mathematics, the curriculum consisted of land surveying, navigation, military engineering, and astronomy.

Philipp Apian continued the Ingolstadt tradition.145 Initially he was taught by his father and a private teacher. In 1537 the duke’s ten-year-old son Albrecht (later Albrecht V of Bavaria), a little older than Philipp, was sent each day to Peter Apian for instruction in cosmography, geography, and mathematics. At the age of eighteen, Philipp was sent on a study trip that led him to Strasbourg, Dôle, Paris, and Bourges, from which he returned shortly before the death of his father in 1552. He was elected successor to his father’s teaching chair, and in 1554 was given the task of mapping Bavaria. By 1561 the surveying, which had occupied the summer months, had been accomplished (Apian taught his courses in the winter months).146

Philipp Apian had to vacate his chairs twice for religious reasons. In 1569 he not only had to leave Ingolstadt, but had to leave Bavaria completely because he had refused to swear the Tridentine profession of faith. That same year he became professor of astronomy and geometry at Tübingen University. When he refused to sign the Formula of Concord in 1583, he was forced to leave the university but was allowed to remain in Tübingen, where he died in 1589.

Apian was, after Stöffler and Scheubel, the third mathematician at the university in Tübingen who took the improvement of maps to heart. Apian’s successor was Michael Mästlin, a professor in Tübingen from 1580, who had been a student of Apian.147 He, in turn, was the teacher of Johannes Kepler and Wilhelm Schickard. Another pupil of Mästlin’s during the years 1597–1602 was the Austrian Abraham von Höltzl, who produced a map of the Schwäbischen Kreis about 1620.148

Schickard had studied in Tübingen and was appointed Mästlin’s successor as professor of mathematics in 1631, having been professor of Hebrew since 1619.149 He was the fifth Tübingen mathematics professor to have cartographic interests. In 1624 he had begun a systematic record of the land of Württemberg because he was dissatisfied with the maps available. After his early death from the plague, the complete and final drawings of thirteen map sheets are said to have been sent to Amsterdam for printing. Only sheet eight, showing Tübingen and the surrounding area, is still in existence.150

Erasmus Reinhold (d. 1574) was the little-known son of the Wittenberg astronomer of the same name, who died at the early age of forty-two in 1553.151 He came from Saalfeld in Thuringia,152 where his son also set up business as a doctor. It is probable that he studied under his father in Wittenberg. From Reinhold we have the tract mentioned earlier as well as maps of the districts of Altenburg and Eisenberg in Thuringia.

The cartographer Tilemann Stella from Siegen also studied in Wittenberg before going to Marburg for two years. He returned to Wittenberg and Cologne between 1546 and 1551, before he was summoned to Schwerin to the court of the duke of Mecklenburg as a mathematician and geographer with the special responsibility to determine boundaries. From 1582 until his death in 1589, he lived at the court of the Count Palatine bei Rhein in Zweibrücken. In 1560 he carried out angle measurements from the tower of St. Stephan’s Cathedral in Vienna to the most prominent features of the city. He confided this only in his diary. His maps have been executed as oil paintings on canvas; they have been neither copied nor thoroughly studied concerning their accuracy.153

Daniel Schwenter studied mathematics from 1602 at Altdorf University, which was a part of Nuremberg territory, under Johannes Prätorius. Already as a pupil at school in Sulzbach, he had shown a special interest in geometry and had studied, among other subjects, Hirschvogel's geometry from 1543. In 1608 he became professor of Hebrew studies at Altdorf, and in 1628 he obtained the chair of mathematics. The later Regensburg accounting teacher Georg Wendler, who had studied in Altdorf under Schwenter's successor Abdias Trew (from 1636 professor of mathematics), wrote of practical terrain exercises in which he and Trew made sightings of Altdorf from a nearby hill and carried out measurements.

Augustin Hirschvogel came from Nuremberg and served an apprenticeship as glass painter under his father, Veit. Nothing is known about his school education, but it seems clear that he must have attended at least one of the many Nuremberg Rechenschulen on the evidence of his later achievements. In 1543 he published a Geometria, paying particular attention to the ideas of perspective. In 1547, after doing other cartographic work, he produced a city plan of Vienna based on measurements. In 1552 he was writing about his geometrical methods and the various instruments used and finally earned the honorary title of “Mathematicus” in Vienna. He must have become familiar with instrumentmakers and their products while in Nuremberg; the instructions contain one of the earliest known descriptions of a plane table. Because Hirschvogel never claimed to be the inventor of this instrument, it is assumed that he took knowledge gained in Nuremberg with him to Vienna.

Paul Pfning, too, came from Nuremberg, where he was a merchant. He began to study in Leipzig in 1562 at the age of eight—the University at Altdorf was not opened until 1575—and in 1594 presented his hometown with the atlas that he had produced—as well as other maps—himself. The accuracy of Pfning's maps has not yet been studied. He is the only cartographer of the period under study who carried out cartography more or less as a hobby.

Although born and raised in Windsheim, Sebastian Kurz later went to Nuremberg to become a teacher. He attended only the Rechenschule in Windsheim and later became principal of this school for a short time. In 1617 he published a Tractatus geometricus, having already translated a book on the practice of surveying (Practica des Landvermessens) as well as descriptions of instruments from Dutch in 1616. The author of the Practica (original title, Practijk des lantmetens) was surveyor Jan Pietersz. Dou of Leiden.

The vast majority of the authors on surveying during the sixteenth and seventeenth centuries had university studies behind them and had become university profes-
University of Leipzig, but probably never studied there. Abraham was instructed by his father, and after the latter’s death in 1559 he took over the management of the Rechenschule in Annaberg in the Erzgebirge as well as the office of recorder (of the owners of mining shares) for the elector. In 1559 he created a map of the Obererzgebirgischen Kreis based on measurements he had carried out himself and in 1575 created another of the Vogtländer Kreis. Both areas were newly added territories of Saxony. The primary objective of Ries in carrying out the measurements was to determine the size of the new districts. A gold-plated multipurpose instrument with the basic form of an astrolabe, produced by Abraham Ries, was kept in the elector’s Kunstkammer and could still be found there in 1874.

At the same school was Lucas Brunn from Annaberg. He later studied in Leipzig and Altdorf (under Prätorius) and obtained his master’s degree. He constructed precision instruments and was very possibly the inventor of the adjustable-screw micrometer. In 1619 he became “Inspektor” of the Dresden Kunstkammer, and in 1625 he published Euclides Elementa practica.

A complete exception to the university background of authors of surveying manuals was Nicolaus Reimers of the illustrious lineage “de Baren” (Ursus) of Dithmarschen. He first learned to read and write at the age of eighteen, but due to his evident talent he was chosen to serve Heinrich von Rantzau, the Danish governor of southern Schleswig-Holstein in 1573/74. One of Rantzau’s jobs was to survey the property, a task carried out for tax purposes and for which it was important to calculate areas very carefully. Whether the surveying in fact led to the construction of a map is not known, but there must have been this intention at least, because in 1583 Reimers had a surveying manual printed to which he gave the title Geodæsia Rantzoviana, which described the duties of the ancient agrimensores and the measuring of field area.

Although various triangulation and astronomical methods were the primary means used to survey land, there were also the methods of the ancient agrimensores, but this second means of surveying the land was seldom seen in Germany until the end of the sixteenth century. However, it is evident in the book Fundamentum geographicum, a teaching manual on the construction of maps that Caspar Dauthendey published in 1639, shortly before his death. He taught geometry and geography and drew and published a map of Brunswick (Braunschweig) on the basis of his own mathematical observations. In his work he complained of the difficulties of land surveying, which was more important than ever after the destruction caused during the Thirty Years War. He especially noted that the field surveyor was unqualified due to a lack of proper geometrical knowledge, while the somewhat higher-ranked geometers shrank from carrying out the work of measuring. He therefore suggested that the field surveyors be provided with better geometrical education.

The German empire of the Middle Ages was constituted as a Personenverbandsstaat (state as an association of persons), in which privileges and rights were not connected with the possession of land but rather awarded to individuals. Therefore, field surveying in the ancient tradition made little sense.

In England, the profession of “surveyor” had been known for a longer time, and its legal interest came from large land distributions after the dissolution of the monasteries. The surveyor’s task was the administration of large estates and their supervision, for which an overview of the extent of the land was necessary. For this reason, one finds pictorial representations that—depending on quality, scale, and format—can be regarded as maps. Christopher Saxton and John Norden were primarily “surveyors” who were also busy producing maps. We know little about Saxton’s education. As a young man, from about 1554 to 1570 he was a servant to the Dewsbury clergyman John Rudd, from whom it is likely that he received his training. Rudd himself had made journeys in 1561 in order to obtain information for the construction of maps, but nothing is known about his methods, and Rudd’s maps have not survived. It is assumed that Saxton accompanied him. Until about 1587, Saxton was always someone’s servant, which makes his lifestyle completely different from those of the German mapmakers. From 1587 he worked for himself as a “surveyor.”

166. Zinner, Deutsche und niederländische astronomische Instrumente, 266, and Wunderlich, Kurzachsische Feldmesskunst, 130–35.
168. Launert, Nicolaus Reimers, 134–45.
171. Tyacke and Huddy, Christopher Saxton, 24, and Ior M. Evans and Heather Lawrence, Christopher Saxton: Elizabethan Map-Maker (Wakefield, Eng.: Wakefield Historical Publications and Holland Press, 1979).
173. Tyacke and Huddy, Christopher Saxton, 24.


**Links between Surveying and Maps**

We know from only very few contemporary accounts which methods and instruments were used to construct the many maps produced in the period covered by this chapter. In many cases, a quantitative study of accuracy tells us more about a map than all the earlier or modern praise or criticism. However, not all maps are suitable for an analysis of their exactness. Without going into detail as to which scale should be set as an upper limit, it is absolutely clear that for world or continent maps—despite all the obvious differences—the question of their relationship to astronomical-geometrical surveying in the early modern age is problematic. General maps produced during this time, such as those of the larger countries like France, Italy, and the German empire, should not be included in the discussion. None of these countries was measured terrestrially during the early modern era, and the number of locations with reliable coordinates was much too small. Bavaria, which in those days extended only as far north as the Danube and westward to the river Lech, was the largest state on the Continent to be mapped in the sixteenth century on the basis of astronomical and geometrical methods. All other newly produced maps with claims to accuracy were of smaller areas. Only by keeping this background in mind is it worthwhile considering the question of precision. And only then can the question of establishing a link between the previously mentioned surveying techniques and instrumentation and the maps themselves be posed.

Geographical drawings and even paintings are preserved in the larger cartographic collections, for which an analysis of accuracy is neither possible nor indeed worthwhile. They were produced during the early modern period as an *Augenschein* (a kind of eyewitness evidence) of a given landscape. For example, a draftsman who was sworn in by a court would have captured the situation involving a legal dispute over land, so the court could gain an impression of the situation without having to examine the location personally and without having to rely on the biased information of one of the parties involved. Never intended as a plan view, but often with several changing perspectives, the landscape represented by the draftsman’s drawing lacks any geometrical basis. Other “maps,” too, such as those in the *Cosmography* of Sebastian Münster, dispensed with geometrical fundamentals as well as arrangement in a grid system. One cannot apply the standard of accuracy to these sketches.

The question of accuracy is also problematic in changing landscapes. In Ostfriesland, for example, the inhabited area was, in early modern times, only a very narrow ring around a huge moorland. Beyond was the sea, which in those days had anything but an exact boundary. The maps of the astronomer David Fabricius from 1589 and 1592 were greatly improved on by the mathematician Ubbo Emmius in 1595, but the severe storm tides of 1625 made his results largely invalid. In 1627 Johann Conrad Musculus recorded the immense damage done in his map of the dikes. Despite having experience in surveying, he never achieved anything like the quality obtained by Emmius and Fabricius.

Another reason to avoid assessing a map from the early modern era by quantitative methods lies in the great differences in scale within one sheet. For example, districts or towns and cities that were important to the mapmaker or the client were sometimes drawn to a larger scale than other areas, because there was more information in populated areas that was worth showing than in the surrounding area, where there was little of interest to the mapmakers’ contemporaries—neither roads nor destinations. This was the case with the maps of England by Christopher Saxton. These changes in scale, however, did not lead to a general distortion of the area represented in Saxton’s atlas of England and Wales. Vector analysis of the coordinates of approximately sixty towns shows that the geographical latitude has been measured very well on the whole. When it comes to longitudes, Cornwall and Wales stretch too far toward the west, but others are amazingly good, even if the variations are in different directions. As Skelton wrote, one would like to know whether this result was based on astronomical observations, because the accuracy cannot be coincidental.

Today not only can one answer in the affirmative, but from this precision it is also possible to name the methods that were applied. Because the lunar eclipse method entails an inaccuracy of some twenty to thirty minutes of

time in establishing the exact start and end of full eclipse, which represents five to eight degrees of longitude at a particular latitude, the spread of error would have to be much larger if Saxton had used this method. And Saxton could not have taken the coordinates from anyone else. In 1574—the same year that Saxton began collecting his information on the ground—William Bourne published a list of coordinates in which only London has a relatively exact value; other cities such as Hereford, Oxford, Cambridge, and Bourne’s hometown, Gravesend, vary considerably from Saxton’s values.179 Against this background, it becomes clear why Saxton included a border scale for his wall map, which he divided into ten-minute intervals. As opposed to Apian’s use of astronomically determined points and applied trigonometry to fill in a single-scale Landtafeln, Saxton chose to depict towns in greater detail than the surrounding areas, with corresponding differences in scale, so it is hard to imagine that Saxton could have carried out an exact terrestrial survey, more or less according to the geometrical methods of Gemma, even if, as in the case of Wales, he was issued a pass by which he was permitted access to towers, castles, high-lying locations, and hills so he could see the lay of the land.180 After 1587, Saxton conducted estate surveys; his last known work was completed in 1608.181

Saxton’s contemporary, the “surveyor” John Norden, left a series of maps, but unfortunately none of his methods. No study of his accuracy has been carried out, as has been done for Saxton’s wall map. Norden was different from Saxton inasmuch as the making of maps was for a time his chosen professional aim, but one he had to abandon because payment was not guaranteed.182 He set store by the ability of the user of his maps to ascertain the distance between two towns from them. But that does not permit one to conclude anything about the methods used.

The military engineer Robert Lythe carried out surveying work in southern Ireland in the years 1567–70 and on the basis of his efforts drew up a map that included no coordinates.183 One learns little of his methods from his own writings. He traveled mainly by boat on Irish rivers, and he presumably measured latitudes and longitudes; he could not get the necessary overview for terrestrial surveying from a boat.184 He expressly vowed to work according to the “rules of cosmography.” Thus, one is surprised at the lack of coordinates on his map. However, he did carry out measurements approximately every five miles, providing the English administration with a much-improved knowledge of the land hydrography.185

One of the methods used by Saxton, Norden, and Lythe in order to learn more about an area in question was to seek the accompaniment and assistance of local guides who could provide them with the names of settlements, river routes, woods, and forests.186 This must have considerably speeded up such surveying journeys. The maps and sketches of these “surveyors” suggest—as far as is known—a readiness to pay attention to detail, revealing a different purpose behind these representations than is inferred from the more generalized style of regional maps.

A similar difference exists between the plans constructed by engineers for irrigation and drainage in parts of Italy (e.g., Venice) and regional maps.187 The so-called periti who specialized in this kind of work had limited terms of office and clearly defined instructions. They did not need any special geometrical training, as shown by the experience of the artist and engineer Cristoforo Sorte from Verona.188 From 1556 to 1564 and from 1589 to 1593 he was perito ordinario, and in this role he produced a large number of drawings of irrigation and drainage systems. He was working for the Camera ai Confini in the years following 1570 and mapped the alpine border region near Venice. Another perito, Giacomo Gastaldi, born in Piedmont, was busy carrying out surveying work on the river Adige between 1550 and 1556.189 These very large-scale plans did not lead to any direct improvement in the construction of even his own published maps. An example of how unrealistically Gastaldi himself illustrated the river system in the Venetian terra firma is a map published without a title shortly after his death, with Padua as its centerpiece.190

The establishment of borders also played a major role for the Grand Duchy of Florence. The Archivio di Stato in Florence has a separate department, the Archivio dei Confini, in which, among other works, there are ten large folio volumes of topographical maps.191 In a map from 1643, Lunigiana (north of Carrara) is illustrated—totally unrealistically—as if the border is formed by a ring of...
mountains around a wide valley basin.¹⁹² This style of representation harks back to the Roman tradition, wherein mountains could be imagined to form borders, which was never adopted in any European mountain area. Detailed topographical knowledge did not, however, necessarily lead to realistic regional maps. The map of Toscana, with a grid in the borders and based on a woodcut by Girolamo Bell’Armato of 1536, which was reprinted by a variety of publishers as a copper engraving until 1646, shows the Tiber and Arno connected by the Chiana, in error.¹⁹³ In reality, today’s highly fertile Chiana Valley was then, in fact, a dreaded swamp area that was first drained dry in the second half of the eighteenth century by canals on which the famous mathematicians and engineers Evangelista Torricelli and Vincenzo Viviani worked. Although the Chiana Valley belongs topographically to the newly founded Academia de Matemáticas, the work was definitely intended for the purposes of internal administration. What is noteworthy is the way that contemporaries valued the geometrical and astronomical principles of maps. If one looks at the mediocre results (i.e., the surviving maps), one ought to take into account the actual enthusiasm of the surveyor—not the standards or the reports. Here is only one characteristic example: Andreas Bureus was the first surveyor and cartographer to serve in the central office of surveying in Sweden (the Lantmäterikon-toret), founded in 1628.²⁰⁰ There was a gaping discrep-

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In addition to the lack of practical application of triangulation, the use of astronomically defined coordinates in map compilation is also striking. Although many of the mapmaking principles known in the Renaissance were based on astronomical applications, such as the method of fixing star positions with coordinates, the imprecision of longitude measurements in a terrestrial context rendered this method impractical. Thus, Münster’s request for information from administrative civil servants and fellow scholars with which to compile the maps for his Cosmography did not generate any information derived from astronomical observations of latitude and longitude. The lack of geodetic control in large-scale maps resulted in a lack of correspondence between manuscript surveyed maps and regional maps intended for publication. Thus, the positional information found on Giacomo Gastaldi’s large-scale maps associated with hydrographical management failed to find its way onto the smaller-scale regional maps that bear his name as compiler.

The bulk of the theoreticians were scholars and academics with a background in mathematics, beginning with the influential Johannes Stöffler, whose treatise on practical geometry was modified by several authors in the early sixteenth century. Later in the century they were joined by more amateur mathematical practitioners in England or by the Rechenmeister (computation teachers) in Germany. The demand for the services of these scholars and practitioners varied across Europe. Many of them taught practical geometry in the context of the general university course of the Quadrivium, while others, such as Philipp Apian, conducted surveying work primarily under princely patronage. Differences in the legal rights associated with land ownership also affected the demand for land surveying.

Just as the extant maps from the period—with very few exceptions (the maps of Philipp Apian and Christopher Saxton, for example)—cannot be used as primary sources to indicate the use of systematic surveys and triangulation, surviving surveying instruments are not reliable guides to the methods that might have been employed or the precision with which they might have been carried out. Many instruments were designed to demonstrate the ingenuity of their makers rather than to be of immediate utility, and thus they were often far too complicated for a surveyor to understand. Thus, despite early precursors of the theodolite, such as the polimetrum or torquetum, this instrument did not find wide use until the eighteenth century.

The overall lack of correspondence between theory and practice in land surveying mirrors a similar lag in the general cartography of the Renaissance, where modern methods of compiling maps had been postulated long before observations of sufficient precision were possible. It was not until the eighteenth century that observational practice was to catch up with the mathematical theory.
